Testing the Self-Consistency of MOND With Three Dimensional Galaxy Kinematics

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ABSTRACT

We propose a technique to test the idea that non-standard dynamics, rather than dark matter halos, might be responsible for the observed rotation curves of spiral galaxies. In the absence of non-luminous matter, a galactic disk's rotational velocity and its vertical velocity dispersion can be used jointly to test the self-consistency of the galaxy's dynamics. A specific illustrative example, using recent measurements of the disk kinematics of M33, shows this to be a promising approach to assess the viability of Modified Newtonian Dynamics (MOND).

Subject headings: Dark Matter—galaxies: kinematics and dynamics—galaxies: individual(M33)

1. INTRODUCTION

Understanding the structure and kinematics of spiral galaxies, in particular explaining their rotation curves at large galactic radii, remains one of the pressing open questions in astrophysics. Optical observations of galactic rotation curves find that rather than falling off as one would expect from galaxy models where the mass traces the observed light, the rotational velocities remain constant at large radii. These findings are further borne out by radio observations of the 21 cm line, from HI gas in the outer parts of the galactic disk. An overview of rotation curves is provided in Sofue & Rubin (2001), Persic et al. (1996) and Borriello & Salucci (2001).

There is strong evidence from the CMB data for considerable amounts of non-baryonic dark matter on the cosmological scale (Spergel et al. 2003). While is it enticing to imagine

that the dark matter problems on the galactic and cosmological scales have a common resolution, this need not necessarily be the case. Our focus in this paper will be on the galactic dark matter problem as manifested in the ubiquitous observation of flat rotation curves.

A number of ideas have been put forth to account for flat rotation curves. All require some form of new physics. These ideas include:

- 1. New particles: Missing mass in the form of galactic dark matter, most likely non-baryonic (Alcock et al. 2000).
- 2. New interactions: Non-gravitational long-range couplings might exist, or gravitational physics might be subject to revisions over large distances.
- 3. New dynamics: scenarios such as Modified Newtonian Dynamics (MOND) in which gravity from visible matter is the only force acting but the system's response takes on new aspects.

The community consensus at present prefers the dark matter hypothesis, but galactic dark matter has thus far evaded all attempts to detect it. We should strive to test, whenever possible, alternatives to the dark matter scenario.

1.1. The MOND Approach to the Rotation Curve Puzzle: Novel Dynamics

Motivated by the observed spiral galaxy rotation curves, Milgrom (1983) proposed a modification of the dynamics of non-relativistic matter. This modified behavior, termed MOND for Modification of Newtonian Dynamics, is conjectured to arise only in the regime of low accelerations. MOND is a proposed modification to an object's acceleration under an applied force, such that $a = g/\mu(x)$ where g is the acceleration expected under Newtonian physics, and $x = a/a_0$ depends upon the MOND acceleration scale $a_0 \sim 1.2 \times 10^{-10} m/s^2$, with $\mu(x \gg 1) \simeq 1, \mu(x \ll 1) \simeq x$. A commonly adopted form is $\mu(x) = x/\sqrt{1+x^2}$. In general a gravitating system's behavior under MOND can be described by taking the Newtonian description and replacing G, the coupling constant, by G/μ , with the understanding that the dynamics is being altered rather than the nature of the gravitational interaction.

In this scenario the mass of a galaxy resides in the ordinary astronomical components that we can detect by their emission or absorption of electromagnetic radiation, and the galaxy's light distribution traces out its mass distribution. A review of MOND as an alternative to dark matter is presented in Sanders & McGaugh (2002).

In the MOND model the response of a test particle to an applied force depends upon the magnitude of its absolute acceleration relative to a preferred frame, taken to be the local rest frame of the microwave background. "Overacceleration" at low values of x then produces the observed rotation curves of spiral galaxies. MOND thereby eliminates the need for dark matter, at the expense of novel dynamics at low accelerations. MOND does a remarkably good job of fitting the rotation curves of galaxies across a wide range of surface brightness, with a_0 as the single free parameter (Sanders & McGaugh 2002).

The MOND idea was recently placed on a more formal footing (Beckenstein 2004), but our approach will be cast in the original phenomenological framework, in terms of $\mu(x)$. From this standpoint, the formulation described above suggests an observational test for the self-consistency of MOND. Because $\mu(x)$ depends on the (scalar) magnitude of a particle's total acceleration, comparing the vertical and rotational dynamics of test particles in the disk of a spiral galaxy provides a means to test for self-consistency. This paper proposes a framework for carrying out such a test and illustrates the technique with recent data from M33 (Ciardullo et al. 2004).

Other recent efforts to investigate the viability of the MOND hypothesis using kinematics include using galaxy clusters (Pointecouteau & Silk 2005) and globular clusters of stars (Baumgardt, Grebel & Kroupa 2005). Our approach differs in that, as discussed in the following section, we are checking the self-consistency of MOND rather than comparing the observed kinematics to a prediction.

2. USING 3-d DISK KINEMATICS TO TEST SELF-CONSISTENCY

We will adopt the common model for a galactic disk as a mass distribution with a volume matter density $\rho(r, z)$ given by

$$\rho(r,z) = \rho_0 e^{-r/R_0} \operatorname{sech}^2(z/z_0) \tag{1}$$

where R_0 and z_0 are characteristic scale lengths in the radial and vertical directions. In the MOND scenario, the rotational speed of a thin galaxy with a mass structure described by equation (1) obeys (adapting the expression for Newtonian physics from Padmanabhan (2002))

$$v^{2}(r) = a(r) \ r = \frac{g(r)}{\mu_{r}(r)} r = \frac{4\pi G \Sigma_{0} R_{0}}{\mu_{r}(r)} y^{2} [I_{0}(y) K_{0}(y) - I_{1}(y) K_{1}(y)]$$
 (2)

where a and g are the MOND and Newtonian accelerations respectively, G is Newton's gravitational constant, R_0 is the disk scale length, Σ_0 is the surface mass density at r = 0,

 $y = r/2R_0$ is a dimensionless radius variable, $I_n(y)$ and $K_n(y)$ are n^{th} order modified Bessel functions, and μ_r gives rise to an object's modified response in the radial direction.

The vertical kinematics of the objects in the disk are described by $\sigma_z^2(r) = 2\pi G \Sigma(r) z_0/\mu_z(r)$, where $\Sigma(r)$ is the local surface mass density, z_0 is the vertical scale height, and μ_z accounts for MONDian behavior in the vertical direction. Given these two expressions and the exponential radial form for $\Sigma(r)$ given in equation (1), we can construct the dimensionless "Consistency Parameter" ratio $CP(r) = \mu_r(r)/\mu_z(r)$.

Regardless of the local dynamical law that describes a star's response to feeble forces, CP(r) should be unity, at all radii. This is true even if the system makes a transition from the Newtonian to the MOND regime.

The expressions given above allow us to write CP(r) as

$$CP(r) = \frac{\mu_r(r)}{\mu_z(r)} = 2\frac{R_0}{z_0(y)} \frac{\sigma_z^2(y)}{\sigma_c^2(y)} [y^2 e^{2y} (I_0(y) K_0(y) - I_1(y) K_1(y))]. \tag{3}$$

For a disk-only system, the combination of the observables on the right side should equal to unity at all radii in both the Newtonian (where $\mu=1$) and MONDian (where $\mu < 1$) regimes. CP(r) can be understood as a parameter testing whether the tracer material is behaving in a self-consistent fashion. Note that this expression is independent of both the mass-to-light ratio of the disk material and of the central surface density of the disk.

Ideally, one would obtain both face-on and edge-on observations of a single galaxy. This would then allow the measurement of the rotation curve, of the vertical velocity dispersion, and of the scale lengths R_0 and z_0 . In practice, this is of course not possible.

There is, nevertheless, a realistic possibility of measuring the radial dependence of CP(r), using kinematic information alone. A nearly face-on bulgeless spiral galaxy would provide a powerful testbed. The apparent (projected) rotation curve would be suppressed by an unknown sin(i) factor, where i is the inclination angle, giving an observed $v_{obs} = v_c sin(i)$. This would simply rescale CP by an overall multiplicative factor, so that while still radius–independent it will differ from unity by $1/sin^2(i)$. Given that typical rotational velocities are a few 100 km/s while velocity dispersions are tens of km/s, a tenfold suppression of the rotation curve is quite tolerable. This corresponds to using systems with inclination angles as small as a few degrees. The advantage of using a face-on galaxy is that the line-of-sight velocity dispersion is a clean measure of vertical velocity dispersion, uncontaminated by other components of the velocity ellipsoid within the galactic disk.

The main observational challenge is obtaining high signal-to-noise measurements and then extracting both the circular velocity field and the vertical velocity dispersion. This is a tractable problem. The projected circular velocity produces a Doppler-shifted centroid of a spectral feature. The velocity dispersion can be determined from either the broadening of spectral lines, or by the width of the velocity distribution of a set of individual resolved objects.

We emphasize the fact that as long as the disk scale height z_0 is independent of r, and the self-supporting stellar disk dominates $\rho(r, z)$, only velocity data are needed to test for variation in CP(r).

By selecting nearly face-on galaxies for this test, we lose the ability to measure their vertical scale height and must instead appeal to a statistical argument that invokes measurements of edge-on analogous systems. Measurements of the light distribution of edge-on galaxies in the near infrared, using images from the 2Mass survey (Bizyaev & Mitronova 2002) indicate that for typical galaxies the vertical scale height is independent of galactic radius, with typical values of z_0/R_0 varying between 0.1 and 0.4. We will therefore adopt the working hypothesis that z_0 in equation (3) is independent of r.

3. AN ILLUSTRATIVE KINEMATIC SELF-CONSISTENCY TEST USING M33

There are a few instances where a galaxy disk's vertical velocity dispersion has been measured (e.g. Bottema (1993)). A recent data set on the kinematics of M33 (Ciardullo et al. 2004) provides an interesting test case. These authors obtained line-of-sight velocity data on 140 planetary nebulae in M33. This archival data set provides an opportunity for a concrete example of the CP-violation test outlined above.

3.1. The Kinematic Properties of M33

This local group galaxy is inclined at 56 degrees to the plane of the sky. Various determinations (using multiple techniques) yield a distance modulus of 24.8 ± 0.1 mag. The radial scale lengths for light are $R_0^V = 2.5$ kpc and $R_0^K = 1.56$ kpc in the V and K bands, respectively. This implies (taking the 2Mass-derived typical values for R_0/z_0) a likely vertical scale height z_0 in M33 of a few hundred parsecs.

The M33 inclination angle of 56 degrees is not optimal for our purposes, but Ciardullo et al (2004) provide their best estimates of M33's rotational velocity and vertical velocity dispersion as a function of galactocentric distance. Their results are presented in Table 1. The M33 circular velocity at r=10 kpc implies, in the MOND scenario, that objects at that

radius should experience a threefold increase in their radial (and hence vertical) acceleration, relative to the Newtonian value. The typical acceleration component normal to the disk is $a_z \sim \sigma_z^2/z_0$, well into in the MOND regime. It is therefore the radial component that determines the kinematics of the objects in the disk. An extended rotation curve for M33 is presented in Corbelli & Salucci (2000).

The observational data in Table 1, in conjunction with equation (3), provide us with the opportunity to map out the radial dependence of the consistency parameter, CP, across the face of M33.

3.2. A Mass-Traces-Light MOND Consistency Analysis

In the MONDian view, the ordinary astronomical inventory of M33, plus novel dynamics, produce the observed rotation curve. The light distribution across M33 should then trace the galaxy's mass distribution. We have evaluated the radial dependence of CP using values of R_0 obtained from visible and near-IR wavelengths.

The measurements of M33's kinematics from Table 1 were used in conjunction with equation (3) to determine CP(r), for different values of the structural parameters R_0 and z_0 . One choice we made was to set $R_0 = 1.56$ kpc and $z_0 = 0.4$ kpc, corresponding to roughly the midpoint of the 2Mass aspect ratio distribution. The resulting CP(r) values are shown in Figure 1. Since for the M33 data R_0 is better constrained than z_0 we also explored values of z_0 that produced the CP(r) curve closer to unity, by fitting for z_0 while minimizing the sum $\Sigma(1-CP(r))^2$ for the radii listed in Table 1. With R_0 fixed at 1.56 and 2.5 kpc, the best-fit values of z_0 were 0.23 and 0.079 kpc, respectively, corresponding to values of z_0/R_0 of 0.14 and 0.03. The best-fit value of z_0 for the larger disk scale length is only 80 pc, and corresponds an unreasonably thin disk. The corresponding CP(r) curves for these cases are also shown in Figure 1.

The curves in Figure 1 indicate that, as shown in equation (3), the value of R_0 determines the *shape* of the CP(r) curve, and the z_0 parameter only provides an overall multiplicative scaling that can be adjusted to drive the average CP value towards unity.

In this mass-traces-light analysis, the CP parameter changes by a factor of 3–10 (depending upon the fit used) between the inner and outer regions of the M33 disk. In this illustrative example the CP(r) behavior appears inconsistent with the MOND scenario. The sense of the discrepancy, with CP values less than one, corresponds to $\mu_z > \mu_r$, so that matter appears to be overaccelerating in the radial direction more than in the vertical direction.

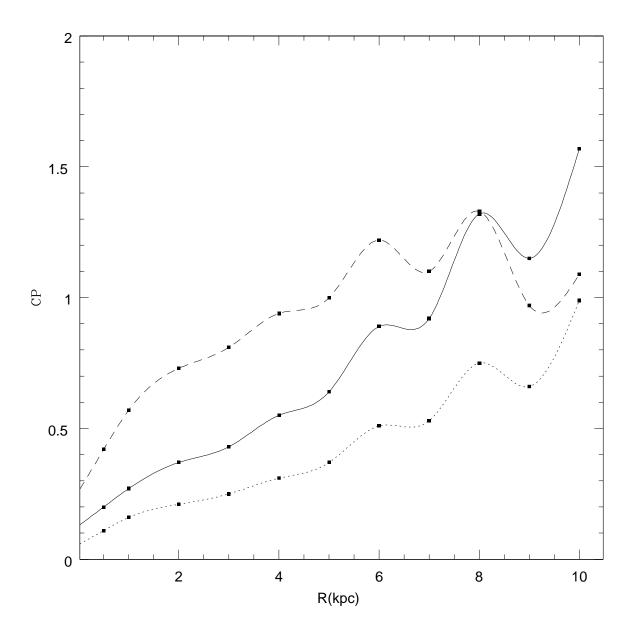


Fig. 1.— Values of the dimensionless Consistency Parameter, CP, in M33 vs. galactic radius, for light-traces-mass scenarios. The plot shows the radial dependence of $CP = \mu_r/\mu_z$, the ratio of the MOND overacceleration parameter in the radial and vertical directions. The curves show CP(r) for galactic structural parameters (in kpc) of (R_0, z_0) equal to (1.56, 0.23)=solid, (1.56, 0.4) = dotted, and (2.5, 0.079) = dashed. A self-consistent MOND galaxy would have CP = 1 at all radii.

3.3. Relaxing the Mass-Traces-Light Constraint: Allowing a Wider Range of R_0 and z_0

The exponential scale length of the M33 disk emission depends upon the passband used to measure the surface brightness, ranging from 1.56 kpc in the K band to 2.5 kpc in the V band. Stepping back from the light-traces-mass approach, is interesting to explore what combinations of R_0 and z_0 would produce a CP(r) closest to unity.

Allowing both R_0 and z_0 as free parameters, with no constraints and with no assumption about the galaxy's mass-to-light ratio, the values that best match CP = 1 are $R_0 = 5.4$ kpc and $z_0 = 0.035$ kpc. This corresponds to a remarkably thin disk, with a vertical scale height of only 35 pc.

The CP(r) profile from this more general fit is shown in Figure 2. This provides a better fit to the kinematic observations, but CP still varies by over a factor of two across the face of M33. Also, the value of $z_0/R_0 = 0.006$ is over an order of magnitude less than typical aspect ratios.

Although we can achieve improved MOND-inspired fits to the kinematic data, these models imply strong radial and vertical gradients in the galaxy's mass-to-light ratio. This is at variance with the elegant what-you-see-is-all-there-is MOND scenario.

4. DISCUSSION

Our objective is to propose a general technique for testing the self-consistency of MOND, using the existing M33 data as an illustrative example. The vertical and circular motions of a galaxy can be jointly used for this test. Potential weaknesses in the argument presented above include i) the assertion that the vertical scale height of galaxies is radius-independent, ii) modeling the galaxy with the form shown in equation (1), and iii) the implicit assertion that either objects overaccelerate, or they don't. The first issue can be addressed with better observations and more statistics, and the second by a more comprehensive treatment of the system's kinematics.

The third concern, namely the isotropy of MONDian dynamics, is an interesting issue. If MONDian behavior arises from a modification of inertia (Milgrom 2005), then this scalar quantity will determine an object's response to any applied force, and it will exhibit the same modified dynamics in all directions. On the other hand one might imagine that MOND only applies component by component, with a modified response only to those forces that would give rise to accelerations below the a_0 threshold. This could produce a difference in the radial

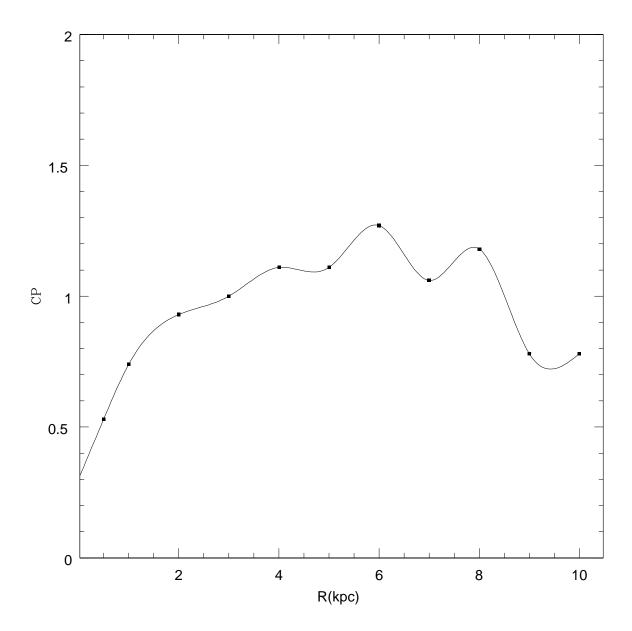


Fig. 2.— Consistency Parameter CP vs. radius after relaxing the light-traces-mass constraint. The solid line corresponds to $(R_0, z_0) = (5.4, 0.035)$ kpc, the best fit to CP = 1 when both are allowed as unconstrained free parameters. This kinematic fit is improved over those shown in Figure 1, but still exhibits significant variation with radius. Also the resulting aspect ratio is at variance with that seen in other similar galaxies.

and vertical dynamics and could perhaps account for a ratio of $\mu_{radial}/\mu_{vertical}$ that differs from unity. In this circumstance however a terrestrial Cavendish experiment conducted at the North or South pole should see differing effective values of G in different regimes of μ .

Sensible next steps to obtaining observations that are optimally suited to the test we propose include 1) assessing the relative merits of planetary nebulae vs. integrated starlight as probes of vertical velocity dispersion, 2) selecting a favorable list of target galaxies, and 3) carrying out a set of appropriate observations. It is sensible to include, as a control, examples of high surface brightness disk galaxies which should have their inner regions in the Newtonian disk-dominated regime where $\mu = 1$, to verify that CP is constant and equal to unity for these systems. H α and 21 cm observations of the velocity field might also contribute to this technique.

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Table 1. Kinematic Properties of M33, from Ciardullo et al. (2004). The first column lists galactocentric distance, the second and third the rotational velocity and vertical rms velocity dispersion, and the fourth column shows the inferred value of μ_r , the radial MOND dynamical parameter, based on the measured circular velocity and radial distance. The circular velocity has been corrected for projection effects and represents the best estimate for the actual rotation curve of the galaxy.

R(kpc)	$v_c \ (km/s)$	$\sigma_z \; ({\rm km/s})$	μ_{radial}
0.5	40	21	0.66
1	55	18.7	0.64
2	80	17	0.66
3	90	14	0.60
4	98	12.5	0.55
5	100	10.5	0.49
6	105	10	0.45
7	106	8	0.41
8	107	7.5	0.37
9	108	5.5	0.34
10	109	5	0.31